48th Annual Dick Schaff Math Superbowl

2023 Level 4 Exam – Secondary Math II Blitz

Directions: (1) Select the most correct answer for each question and mark it on your answer form.

(2) No calculators of any sort are allowed.

(3) Note that N.O.T. means "None of these."

1. The positive x-coordinate of the x-intercepts of $y = 3(x+2)^2 - 48$ is

(a) -7 (b) 6 (c) -5 (d) -6 (e) N.O.T.

2. What is the midpoint of a line segment with endpoints $(-2,3)$ and $(4,6)$?

(a) $\left(1, -\frac{9}{2}\right)$ (b) $\left(-1, \frac{9}{2}\right)$ (c) $\left(\frac{9}{2}, 1\right)$ (d) $\left(-\frac{9}{2}, 1\right)$ (e) N.O.T.

3. If $i = \sqrt{-1}$, then what is the value of $4i^{2023}$?

(a) $4i$ (b) 4 (c) -4 (d) $-4i$ (e) N.O.T.

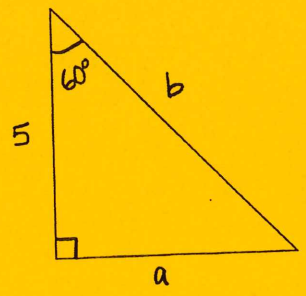
4. Which of these ordered pairs is a solution to the system of equations $\begin{cases} y = 0.6x + 8.6 \\ y = x^2 - 3x + 7 \end{cases}$?

(a) $(4, 36)$ (b) $(0, 1)$ (c) $(1, 5)$ (d) $(-0.4, 8)$ (e) N.O.T.

5. Determine the center of the circle $x^2 + y^2 + 12x - 2\sqrt{30}y - 34 = 0$

(a) $\left(\sqrt{30}, 6\right)$ (b) $\left(12, -2\sqrt{30}\right)$ (c) $\left(-6, \sqrt{30}\right)$ (d) $(6, 30)$ (e) N.O.T.

6. Find the values of a and b in the triangle below.



(a) $a = 5\sqrt{3}, b = 10$ (b) $a = \frac{5}{\sqrt{3}}, b = 10$

(c) $a = 5\sqrt{2}, b = 15$ (d) $a = 5\sqrt{2}, b = \frac{5}{\sqrt{2}}$ (e) N.O.T.

7. Factor $x^4 - 25$ as much as possible.

(a) $(x^2 - 5)(x^2 + 5)$

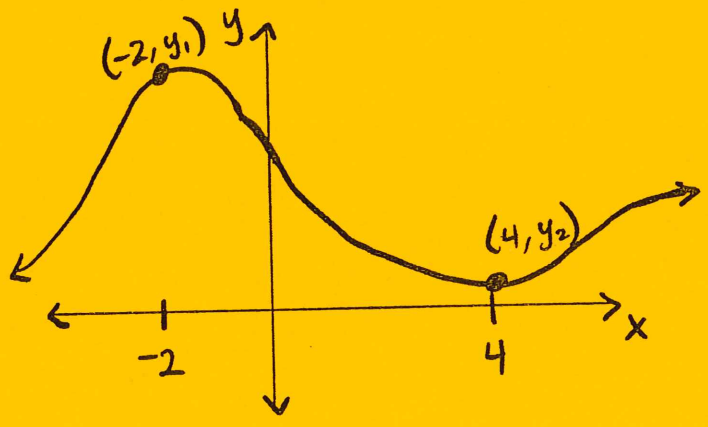
(b) $(x^2 - \sqrt{5})(x^2 + \sqrt{5})$

(c) $(x - \sqrt{5})(x + \sqrt{5})(x - i\sqrt{5})(x + i\sqrt{5})$

(d) $(x + 5)(x - 5)(x + 5i)(x - 5i)$

(e) N.O.T.

8. The graph of the function f is given below. What can be said about the average rate of change of f from $x = -2$ to $x = 4$?



(a) The average rate of change is positive.

(b) The average rate of change is zero.

(c) The average rate of change is negative.

(d) The average rate of change is quadratic.

(e) N.O.T.

9. If the side lengths of a rectangular box are quadrupled, by what factor does the surface area increase?

(a) 4 (b) 2 (c) 16 (d) 64 (e) N.O.T.

10. Consider the function $y = f(x)$. Which of the following would represent the graph obtained by horizontally stretching f by 4 and shifting the resulting graph 10 units to the right and 10 units upward?

(a) $y = f(0.25x - 2.5) + 10$

(b) $y = f(10x - 4) + 10$

(c) $y = \frac{1}{4}f(x + 10) + 10$

(d) $y = f(4x + 40) + 10$

(e) N.O.T.

11. Suppose you spin a spinner that has three colors: red, blue, and green. The probability of not spinning red is 0.40 and the probability of not spinning blue is 0.73. What is the probability of spinning green?

(a) 0.30 (b) 0.33 (c) 0.13 (d) 1.13 (e) N.O.T.

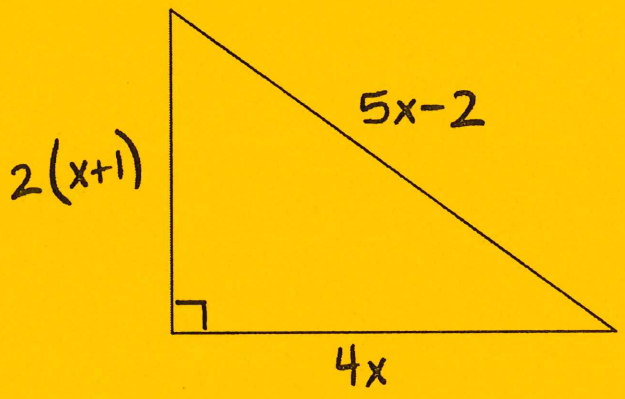
12. Find the solution set of $|x^2 + 3x - 2| = 2$

(a) $\{-4, -3, 0, -1\}$ (b) $\{-4, -3\}$ (c) $\{0,1\}$ (d) $\{4, 3, 0, 1\}$ (e) N.O.T.

13. Simplify $27^{\frac{5}{3}}$ as much as possible.

(a) 243 (b) 7.22 (c) 9 (d) 81 (e) N.O.T.

14. Given the right triangle below, determine the value of x.



(a) 5 (b) 10 (c) 5.6 (d) 12 (e) N.O.T.

15. Simplify $\frac{4+2i}{3-7i}$, where $i = \sqrt{-1}$.

(a) $\frac{1}{29} + \frac{17}{29}i$ (b) $\frac{9}{5}i$ (c) $\frac{4}{3} - \frac{2}{7}i$ (d) $-\frac{1}{29} + \frac{17}{29}i$ (e) N.O.T.

16. If $x = 3 + 2i$ is a root of the quadratic function $f(x) = ax^2 + bx + c$, what is the second root of f?

(a) $3 + 2i$ (b) $3 - 2i$ (c) $9 + 4i$ (d) 3 (e) N.O.T.

17. Suppose the probability a driver is pulled over while speeding and gets a ticket is 0.52. Suppose the probability a driver gets a ticket given that they are pulled over while speeding is 0.65. What is the probability that a driver is speeding?

(a) 0.585 (b) 0.13 (c) 0.8 (d) 1.25 (e) N.O.T.

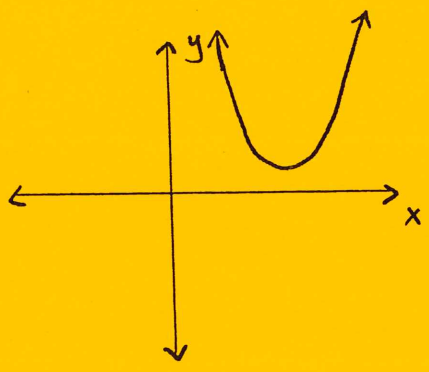
18. Expand the following expression and then simplify as much as possible: $(x^2 - 3)(2x^3 + 2x^2 + 3x + 4)$

(a) $x^3 + 4x + 4x + 2$ (b) $2x^4 - 3x^3 - 2x^2 - 9x - 12$

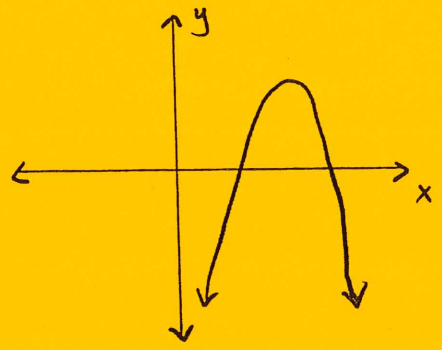
(c) $-11x - 12$ (d) $2x^5 + 2x^4 - 3x^3 - 2x^2 - 9x - 12$ (e) N.O.T.

19. Which of the following would accurately depict the graph of $f(x) = 2 - (x+3)^2$?

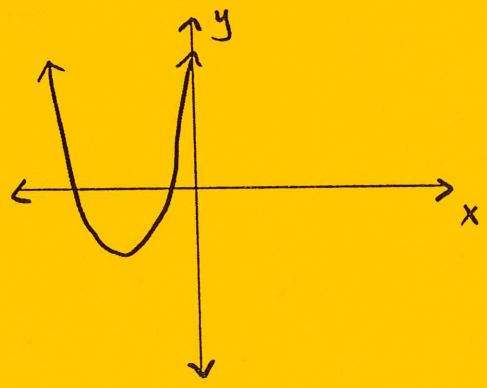
(a)



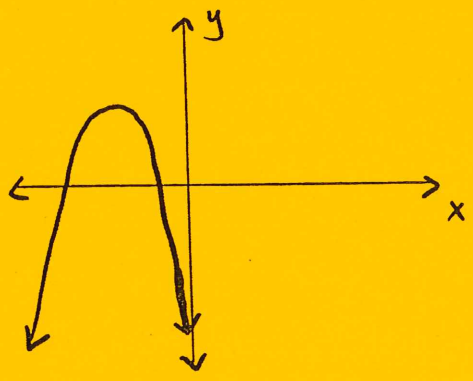
(b)



(c)



(d)



(e) N.O.T.

20. Simplify as much as possible: $(14z^2 - 12z + 4) - (18z^2 - 4z + 2) + (z^2 + 3)(z - 5)$.

(a) $z^3 + 32z^2 - 16z + 4$ (b) $z^3 - 9z^2 - 5z - 13$

(c) $z^3 + 9z^2 - 18z - 12$ (d) $z^2 - 16z - 13$ (e) N.O.T.

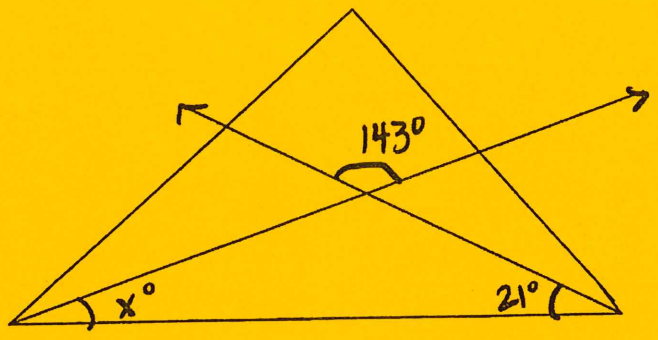
21. For $x, y > 0$, simplify $\left(\frac{9x^2y^{\frac{1}{4}}}{25x^{\frac{2}{3}}y^{\frac{7}{8}}}\right)^{1/2}$ so that only positive exponents occur.

(a) $\frac{9x^{2/3}}{25y^{5/16}}$ (b) $\frac{3x^{2/3}}{5y^{5/16}}$ (c) $\frac{3x^{5/16}}{5y^{2/3}}$ (d) $\frac{9x^{16/5}}{25y^{3/2}}$ (e) N.O.T.

22. In the equation $3x^2 + 13x = 41$, what number must be added to both sides to complete the square on the left side of the equation?

(a) $\frac{169}{4}$ (b) $\frac{9}{4}$ (c) $\frac{13}{36}$ (d) $\frac{169}{6}$ (e) N.O.T.

23. Solve for x in the figure shown.



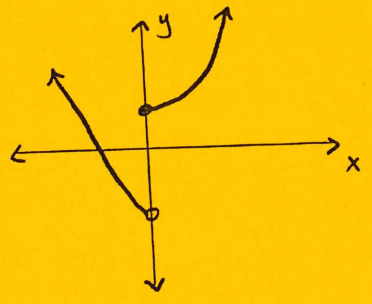
(a) 21 (b) 17 (c) 90 (d) 15 (e) N.O.T.

24. A circle has diameter 17π and center $(3,4)$. What is the area of the circle?

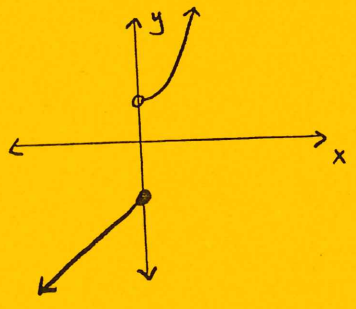
(a) $\frac{289\pi^2}{4}$ (b) $\frac{289\pi^3}{4}$ (c) $\frac{289}{4}$ (d) $\frac{289\pi}{4}$ (e) N.O.T.

25. Determine which of the following would accurately depict the graph of $\begin{cases} -2x + y = -4 & \text{if } x < 0 \\ y = x^2 + 3 & \text{if } x \geq 0 \end{cases}$

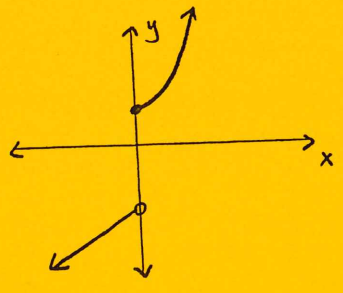
(a)



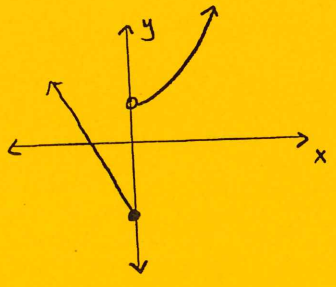
(b)



(c)



(d)

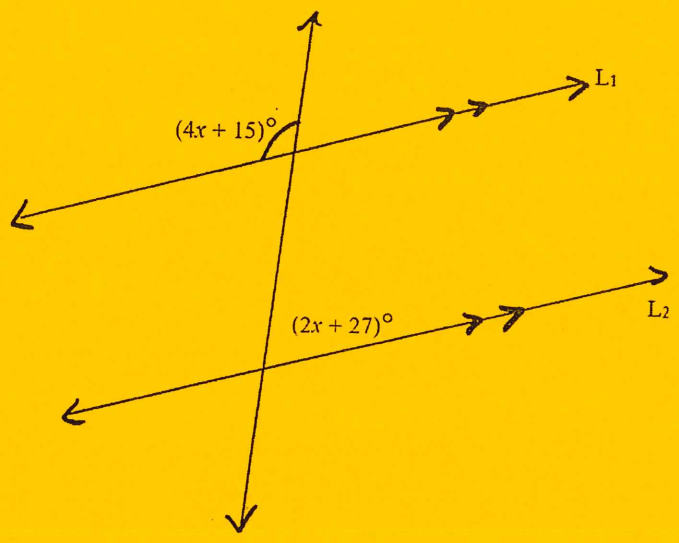


(e) N.O.T.

26. Which of the following lines do not intersect the graph of $y = 3(x-2)^2 - 7$?

(a) $y = -16$ (b) $y = 8$ (c) $x = 3$ (d) $x = -16$ (e) N.O.T.

27. Let L_1 and L_2 be parallel lines, as shown in the figure below. Determine the value of x.



(a) 23 (b) 42 (c) 6 (d) 7 (e) N.O.T.

28. Suppose ΔPQR is an isosceles triangle with $PQ \cong QR$. If the measure of $\angle P$ is one fourth the measure of $\angle Q$, then what is the measure of $\angle R$?

(a) $120°$ (b) $45°$ (c) $60°$ (d) $30°$ (e) N.O.T.

29. What is the maximum value of $f(x) = -7x^2 + 2x - 4$?

(a) $\frac{27}{7}$ (b) 4 (c) $-\frac{7}{27}$ (d) 2 (e) N.O.T.

30. If $f(x) = 3x - 4$ and $g(x) = |2x| - 7$, what is the value of $f(g(-5))$?

(a) -5 (b) 10 (c) 31 (d) 5 (e) N.O.T.

31. Suppose I purchase 40 stamps. Some of these are 44-cent stamps, while the rest are 20-cent stamps. Let x represent the number of 44-cent stamps purchased, and y represent the number of 20-cent stamps I bought. If I spend \$16.40 for all 40 stamps, which of the following systems of equations could be used to determine the number of each type of stamp I bought?

(a) $\begin{cases} 44x + 20y = 40 \\ x + y = 16.40 \end{cases}$ (b) $\begin{cases} 44x + 20y = 40 \\ x + y = 40 \end{cases}$

(c) $\begin{cases} 44x + 20y = 40 \\ x + y = 1640 \end{cases}$ (d) $\begin{cases} 44x + 20y = 16.40 \\ x + y = 40 \end{cases}$ (e) N.O.T.

32. Let $x = 3$ and $x = 11$ be the roots of a quadratic function $y = f(x)$. Which of the following would represent the axis of symmetry of the graph of f?

(a) $x = 7$ (b) $y = 7$ (c) $x = 8$ (d) $y = 8$ (e) N.O.T.

33. In ΔABC, let P be the midpoint of AB, and let Q be the midpoint of BC. Which of the following statements is not true?

(a) PQ is parallel to AC. (b) The length of PQ is half of the length of AC.

(c) ΔABC is similar to ΔPBQ. (d) BP is congruent to BQ. (e) N.O.T.

34. In ΔABC, $\angle A$ is obtuse. Which of the following statements must be true about the sum of the measures of $\angle B$ and $\angle C$?

(a) $m(\angle B) + m(\angle C) < 90°$ (b) $m(\angle B) + m(\angle C) = 90°$

(c) $90° < m(\angle B) + m(\angle C) < 180°$ (d) $m(\angle B) + m(\angle C) = 180°$ (e) N.O.T.

35. What is the volume of a 4-inch tall pyramid with a square base (side length of 6 inches)?

(a) 48 in^3 (b) 60 in^3 (c) 72 in^3 (d) 84 in^3 (e) N.O.T.

36. A circle has diameter 8 π. What is the circumference of this circle?

(a) 8 π (b) 16 π^2 (c) 4 π^2 (d) 16 π (e) N.O.T.

37. A box contains five green balls and four white balls. A sample of two balls is selected at random without replacement. What is the probability that exactly one of the two balls selected is green?

(a) $\frac{5}{6}$ (b) $\frac{1}{20}$ (c) $\frac{2}{9}$ (d) $\frac{5}{9}$ (e) N.O.T.

38. Determine the equation of the parabola with focus at (0,4) and directrix the line $y = -4$.

(a) $y = 16x^2$ (b) $y = \frac{1}{4}x^2$ (c) $y = \frac{1}{16}x^2$ (d) $y = 4x^2$ (e) N.O.T.

39. If ΔABC is similar to ΔEFG, the length of AB is 16 units, and the length of EF is 24 units, then what is the ratio of the measures of side BC to side FG?

(a) 1 : 2 (b) 2 : 4 (c) 1 : 3 (d) 3 : 4 (e) N.O.T.

40. On a right triangle, let θ be an angle between 0° and 90°. If $\sin\theta = \frac{3}{5}$, what is the value of $\cos\theta$?

(a) $\frac{5}{3}$ (b) $\frac{4}{5}$ (c) $\frac{16}{25}$ (d) $\frac{9}{25}$ (e) N.O.T.